Traffic Paradoxes and Economic Solutions

Chengri Ding
Urban Studies and Planning Program, University of Maryland.; Tel: (301) 405–6626; Email: cding@umd.edu

Shunfeng Song [1]
Department of Economics, University of Nevada Reno, NV 89557–0207 USA.; Center for Research of Private Economy, Zhejiang University, Hangzhou, Zhejiang, China.; Tel: (775) 784–6860; Fax: (775) 784–4728; Email: song@unr.edu

ABSTRACT. Previous studies on traffic congestion have emphasized supply-side instruments, such as the expansion of road capacity and improving the management of traffic. However, researchers on transportation have identified several paradoxes in which the usual remedy for congestion—expanding the road system—is ineffective or even counterproductive. This paper presents three paradoxes of traffic flow in their general form and provides economic solutions to overcome them, with an emphasis on demand-side policies by examining the behavior of commuters and using pricing mechanisms.

KEYWORDS. ***Congestion pricing, externality, traffic paradoxes***

1. Corresponding Author

1. Introduction

Urban traffic congestion has been worsening in nearly all major cities around the world, delaying commuters, wasting energy, and causing pollution. To date, most countries have depended on supply-side policies, such as expanding network capacity to mitigate urban congestion. Unfortunately, supply-side policies are often ineffective in reducing urban traffic congestion because urban commuting is subject to "triple convergence." Downs (2004) observed that the expanding capacity has three immediate effects. First, drivers using alternative routes begin using the expanded roads. Second, drivers that had previously been traveling during off-peak costs (either immediately before or after the peak) shift to the peak hours (rescheduling behavior). Third, commuters using public transport begin driving their own vehicles. As a result of triple convergence and induced demand, it appears impossible to remove peak-hour congestion from highways and roads simply by expanding road capacity.

In fact, researchers on transportation have identified a number of traffic paradoxes in which the expansion of a road system to remedy congestion is not only ineffective but also counterproductive under some conditions (Murchland, 1970; Arnott & Small, 1994; Braess, Nagurney & Wakolbinger, 2005). Specifically, the Pigou-Knight-Downs paradox states that expanding the capacity of a road system does not reduce travel cost. The Downs-Thomson paradox states that increasing road capacity can actually exacerbate overall congestion. The Braess paradox proves that adding capacity to a network could in some cases increase the total commuting cost.

Traffic paradoxes exist because commuting generates negative externalities, such as when a single vehicle slows down all vehicles behind it. Thus, in the market equilibrium, drivers tend to drive more, which results in more vehicles on roads and consequently more congestion. To overcome this market failure, tolls must be levied on vehicles to enable the internalization of traffic externalities, allowing social optimization to be reached. The implementation of demand-side remedies to mitigate traffic congestion is highly effective, because price mechanisms not only affect commuting behavior but also generate toll revenue with which the government may improve the transportation network.

This paper examines three traffic paradoxes to prove that expanding a road system to remedy congestion is not only ineffective but often counterproductive. We then show that these paradoxes can be overcome through the implementation of congestion pricing. The main contribution of this work is to prove the three paradoxes and provide socially optimal solutions in a more general form. To our knowledge, no previous study has attempted to accomplish this, except Hartman (2007) who addressed only the Pigou-Knight-Downs. It is our hope that this theoretical analysis will provide a valuable reference for policy makers addressing the issue of transportation.

2. Traffic Paradoxes

This section discusses three traffic paradoxes: the Pigou-Knight-Downs paradox, the Downs-Thomson paradox, and the Braess paradox. They are considered paradoxes because expanding a road system to remedy congestion is ineffective or counterproductive under some conditions. We present these paradoxes in the form of general cases through an analysis of the associated parameters.

2.1 The Pigou-Knight-Downs paradox

The Pigou-Knight-Downs paradox states that expanding road capacity does not reduce travel cost because traffic may simply shift to the upgraded road from other roads, which increases the congestion of the upgraded road.

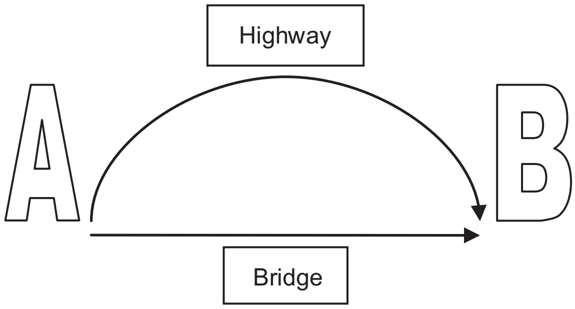

Figure 1. The Pigou-Knight-Downs Paradox

In Figure 1, we assume that a bridge is added to the road from A to B and that the highway is always uncongested. Total travel flow is F, distributed between the bridge (F_1) and the highway (F_2). The average travel cost on the bridge (C_1) is a linear function of the flow-to-capacity ratio and the average travel cost on the uncongested highway (C_2) is a constant. Hence, we have

$$C_1 = a + b\,(\frac{F_1}{R_1})\,; \qquad C_2 = d\,; \qquad F_1 + F_2 = F\,; \qquad (1)$$

where a, b, and d are positive parameters with d > a; is the traffic capacity of the bridge. In equilibrium,

$$C_1 = C_2\,. \qquad (2)$$

resulting in

$$R_1 = bF_1/(d - a). \qquad (3)$$

At the boundary of $F_1 = F$, we obtain the boundary condition for

$$R_1(boundary) = bF/(d - a). \qquad (4)$$

when $R_1 < bF/(d - a)$,

$$\frac{dF_1}{dR_1} = \frac{d - a}{b} > 0, \qquad (5)$$

meaning that increasing the capacity of the bridge will attract more drivers to use the bridge. However,

$$C_1 = C_2 = d\,, \qquad (6)$$

indicating that increasing the capacity of the bridge will not reduce travel cost. The Pigou-Knight-Downs paradox exists for any bridge with a capacity less than $\frac{bF}{d - a}$, because expanding the bridge will only shift travelers to the bridge without reducing travel cost. The Pigou-Knight-Downs paradox reveals that bridge expansion does not necessarily reduce travel cost if at least one person or car is using the uncongested highway.

2.2 The Downs-Thomson Paradox

The Downs-Thomson paradox shows that increasing the capacity of roads could actually increase the overall congestion. This occurs when a shift from public transport causes diseconomies by reducing service frequency or increased fares, leading to a vicious circle in public transit. Ultimately, congestion on the road gets worse and the total commuting cost increases.

In Figure 2, two routes connect A and B. One is a route for private vehicles, with a traffic flow of F_1 . The other is a route for public transit, with a number of passengers of F_2. We assume that the average travel cost on the route for private vehicles (C_1) is a linear function of the flow-to-capacity ratio and the average travel cost on the route for public transit (C_2) has a scale effect.

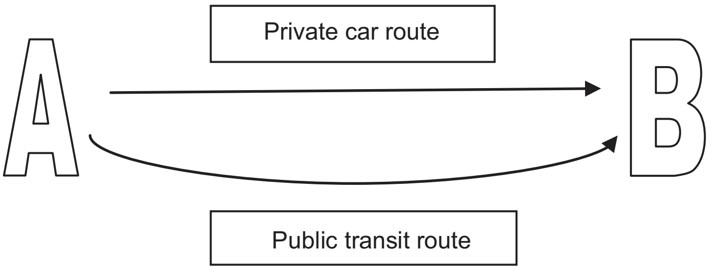

Figure 2. The Downs-Thomson Paradox

Hence, we have

$$C_1 = a + b\,(\frac{F_2}{R_2})\ ; \qquad (7)$$

$$C_1 = d - F_2/\,e\ ; \qquad (8)$$

$$F_1 + F_2 = F\ ; \qquad (9)$$

where a, b, d, and e are positive parameters with $d > a$; R1 is the road capacity of the route for private vehicles; and e describes the scaling effect of public transit. In the equilibrium,

$$C_1 = C_2\ . \qquad (10)$$

we obtain

$$F_1 = \frac{(de - ae - F)}{(be - R_2)}\,R_1, \qquad (11)$$

$$C_1 = a + b\,\frac{(de - ae - F)}{(be - R_1)}\ . \qquad (12)$$

At the boundary of $F_1 = F$, we obtain the boundary condition for R_1

$$R_1\ (boundary) = bF/(d - a) \qquad (13)$$

Within the boundary $R_1 < bF/(d - a)$, because

$$C_1 = C_2 \text{ and } F_1 + F_2 = F, \qquad (14)$$

we have

$$be—R_1 > 0. \qquad (15)$$

Since $F_1 > 0$, given be—$R_1 > 0$,

$$de—ae—F > 0. \qquad (16)$$

which proves,

$$\frac{dC_1}{dR_1} = b(de - ae - F)\ (be - R_1)^{-2} > 0. \qquad (17)$$

Therefore, increasing the capacity of the route for private vehicles within the range of $bF/(d - a)$ will increase travel cost on both routes, proving the Downs-Thomson paradox. The Downs-Thomson paradox suggests that road expansion could be counterproductive because travelers ignore traffic externalities in their modal choices. When they observe the expansion of a route for private vehicles, some switch from taking public transit, which exhibits economic scale, to driving privately, which generates negative externalities. Such a reduction in transit ridership results in a reduction in transit operations and prolonged commuting time.

2.3 The Braess Paradox

The Braess paradox proves that adding capacity to a network can increase the total commuting cost, when the moving entities choose their route according to their own self interest.

In Figure 3, two routes connect A and B before an uncongested causeway is added between

U and W (hence F_3 =0), namely AUB and AWB with traffic flow of F_1 and F_2 , respectively. The segments of AU and WB are congested and travel cost over these conduits increases proportionally with traffic flow. The segments of UB and AW are uncongested and travel cost is assumed to be a constant (*a*).

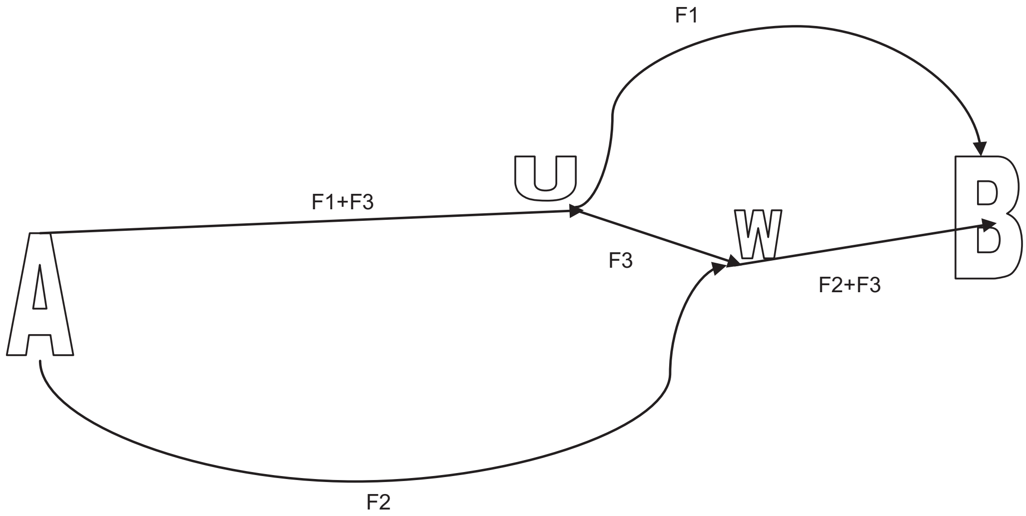

Figure 3. The Braess Paradox

Hence, we have

$$C_1 = a + (\frac{F_2}{e}) ; \tag{18}$$

$$C_2 = a + (\frac{F_2}{e}) ; \tag{19}$$

$$F_1 + F_2 = F ; \tag{20}$$

where a and e are positive parameters. In equilibrium,

$$C_1 = C_2 , \tag{21}$$

which provides the traffic flow on each route (AUB or AWB), half of the total traffic flow, and the average travel cost without the causeway

$$C_{without\ Causeway} = a + \frac{F}{2e} . \tag{22}$$

With the causeway added between U and W, three routes connect A and B, namely AUB, AUWB, and AWB (a fourth route of AWUB would be impractical). In this case, the traffic flow on AU is $F_1 + F_3$ the traffic flow on WB is $F_2 + F_3$ and the traffic flow on the causeway is F_3. Travel cost on the uncongested causeway is assumed to be a constant (k, *with* $k < a$). In equilibrium,

$$C_1 = C_2 = C_3 \tag{23}$$

Where,

$$C_1 = a + \frac{F_1 + F_3}{e}; \tag{24}$$

$$C_2 = a + \frac{F_2 + F_3}{e}; \tag{25}$$

$$C_3 = k + \frac{F_1 + F_3}{e} + \frac{F_2 + F_3}{e}; \tag{26}$$

$$F_1 + F_2 + F_3 = F. \tag{27}$$

We obtain the average travel cost with the causeway and traffic flow distribution,

$$C_{with\ causeway} = 2a - k \tag{28}$$

$$F_1 = F_2 = ke + F - ae; \tag{29}$$

$$F_3 = 2ae - 2ke - F. \tag{30}$$

Therefore,

$$C_{with\ causeway} - C_{with\ causeway} = \frac{F}{2e} - a + k \tag{31}$$

A traffic paradox exists if $F < 2(a-k)e$. In this case, $\frac{F}{2e} - a + k < 0$, meaning that adding the causeway between U and W would increase the average cost of commuting. In other words, expanding the capacity of a network could in some cases increase the total commuting cost. The Braess paradox implies that the construction of a new uncongested highway segment(s) connecting congested highways does not necessary ameliorate the overall traffic situation because this newly constructed causeway attracts users from uncongested highways.

3. The Economic Theory of Congestion Pricing

In the above analysis, commuters make route and modal choices based on commuting costs, i.e., in equilibrium; the average commuting cost is the same for different routes or modals. To put it differently, commuters ignore how much negative externality they cause on other travelers, only paying attention to commuting costs. Therefore, the equilibrium number of commuters for routes and modals is not socially optimal. The theory is presented below.

For each route or modal, let V be the traffic volume and C the average commuting cost. This gives the total commuting cost CV and the marginal social cost,

$$SC = \frac{d(CV)}{dV} = C + V\frac{dC}{dV} = PC + EC \tag{32}$$

where *PC* (the private average cost) is the average commuting cost (*C*) and *EC* is the externality cost ($V\frac{dC}{dV}$). If the average commuting cost increases with the number of commuters, as in the case on congested urban roads, *EC* is positive and social marginal cost (*SC*) is higher than the average private cost (*PC*). Consequently, the equilibrium travel volume (V_E) exceeds the optimal social traffic volume (V_O), i.e., too many commuters are on the road. *VE* is determined according to the average private cost while *VO* is calculated according to the social marginal cost, as shown in Figure 4. If the average commuting cost decreases with the number of commuters, as in the case of public transit and scale economies, *EC* is negative and the social marginal cost (*SC*) is lower than the average private cost (*PC*). Consequently, the equilibrium travel volume (V_E) will be less than the optimal social traffic volume (V_O), i.e., too few passengers using public transit. If the number of commuters does not influence the average commuting cost, then no congestion or externality exists.

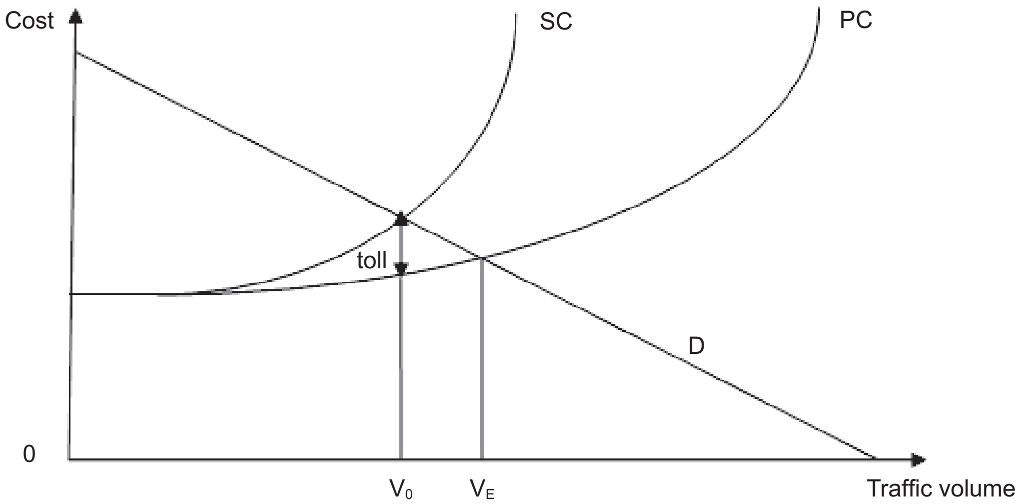

Figure 4. Economics of Congestion Pricing

To attain social optimization, externality should be internalized. In the case of congested urban roads, this suggests that commuters be charged a toll of $V\frac{dC}{dV}$. Because $V\frac{dC}{dV}$ depends on the volume of traffic, the toll should be higher for more congested roads or periods than that for less congested roads or periods. The optimal toll revenue equals $V^2 \frac{dC}{dV}$ and is determined at V_O.

4. Congestion Pricing: Resolving Traffic Paradoxes

In the following discussion, we determine traffic volumes for routes and modals by minimizing the total social cost. If we let the marginal social cost be the same for different routes, then the same solutions will be obtained. For all three traffic paradoxes discussed in Section 2, with congestion pricing, our solutions show that increasing traffic capacity will always decrease the social total cost. Therefore, these paradoxes can be solved by internalizing commuting externality and implementing optimal congestion pricing.

4.1 The Pigou-Knight-Downs paradox

As specified earlier for the Pigou-Knight-Downs paradox, we have

$$C_1 = a + b\left(\frac{F_1}{R_1}\right); \qquad C_1 = d; \qquad F_1 + F_2 = F; \tag{33}$$

where a, b, and d are positive parameters with $d > a$; R_1 is the traffic capacity of the bridge. Accordingly, the total cost can be written as a function of F_1 after considering the flow constraint,

$$TC = C_1F_1 + C_2F_2 = dF + (a - d)F_1 + \frac{b}{R_1}\mathrm{F}_1^2 \tag{34}$$

Minimizing the total cost with respect to F_1, we obtain the following optimal social traffic flow on the bridge, the total social cost, and the relationship between total social cost and bridge capacity:

$$F_1 = \frac{d - a}{2b} R_1 \tag{35}$$

$$TC = Fd - \frac{(d-a)^2}{4b} R_1 \tag{36}$$

$$\frac{dTC}{dR_1} = -\frac{(d-a)^2}{4b} < 0 \tag{37}$$

The above solutions provide two conclusions. First, the optimal social travel flow is half of the equilibrium flow derived in Section 2, demonstrating that the bridge was indeed over-used in the equilibrium when travelers selected routes according to their own private cost. Second, the total travel cost decreases with bridge capacity. Hence, the Pigou-Knight-Downs paradox is solved when traffic flows are distributed by minimizing the total social cost.

At the social optimum, the difference between the marginal social cost and the average private cost, i.e., the traffic externality, will be bF_1/R_1, as evaluated according to the socially optimal traffic flow. This determines the optimal toll and toll revenue on the bridge, which equals

$$Toll = \frac{d-a}{2} \tag{38}$$

$$\text{Revenue} = \frac{(d-a)^2 R_1}{2b} \tag{39}$$

Therefore, to attain a socially optimal solution and solve the Pigou-Knight-Downs traffic paradox, every commuter who is using the bridge should be charged a toll of $(d\text{-}a)/2$. By charging such a toll, travel externality is internalized and some of the commuters will be discouraged to use the bridge, increasing the efficiency of the road network. With the toll at the optimal level, it is not difficult to prove that commuters using the two different routes end up paying the same marginal commuting cost, because

$$C_1 + Toll = a + b\left(\frac{F_1}{R_1}\right) + \frac{d-a}{2} = d. \tag{40}$$

4.2 The Downs-Thomson paradox

As previously specified for the Downs-Thomson paradox, we have

$$C_1 = a + b\left(\frac{F_1}{R_1}\right); \tag{41}$$

$$C_2 = d - F_2/e; \tag{42}$$

$$F_1 + F_2 = F; \tag{43}$$

where a, b, d, and e are positive parameters with $d > a$; R_1 is the traffic capacity of the route for private vehicles; and e indicates the scale effect of public transit. Accordingly, the total cost can be written as a function of F_1 after considering the flow constraint,

$$TC = C_1F_1 + C_2F_2 = dF - \frac{F^2}{e} + \left(a + \frac{2F}{e} - d\right)F_1 + \left(\frac{b}{R_1} - \frac{1}{e}\right)F_1^2 \tag{44}$$

Minimizing the total cost with respect to F_1, we obtain the following social optimal travel flow on the bridge, the total social cost, and the relationship between the total social cost and the capacity of the route for private vehicles.

$$F_1 = \frac{de - ae - 2F}{2(be - R_1)} R_1 \tag{45}$$

$$TC = Fd - \frac{F^2}{e} - \frac{(de - ae - 2F)^2}{4e(be - R_1)} R_1 \tag{46}$$

$$\frac{dTC}{dR_1} = -\frac{b(de - ae - 2F)^2}{4(be - R_1)^2} < 0 \tag{47}$$

The above solutions provide two conclusions. First, because $d > a$, we can easily prove that the social optimal travel flow of private vehicles on the route is smaller than the equilibrium flow derived in Section 2, demonstrating that the route for private vehicles was indeed over-used if route choices are made based on private cost. Second, the total travel cost decreases with the capacity of the route for private vehicles. Hence, the Downs-Thomson paradox is solved when traffic flows are distributed by minimizing the total social cost.

At the social optimum, the difference between the marginal social cost and the average private cost, i.e., the traffic externality, will be bF_1/R_1, as evaluated at the social optimal traffic flow. This determines the optimal toll and toll revenue on the route for private vehicles, which is equal to

$$Toll = \frac{bF_1}{R_1} = \frac{b(de - ae - 2F)}{2(be - R_1)} \tag{48}$$

$$\text{Revenue} = \frac{bF_1^2}{R_1} = \frac{bR_1(de - ae - 2F)^2}{4(be - R_1)^2} \tag{49}$$

Therefore, reaching a socially optimal solution and resolving the Downs-Thomson traffic paradox require that every commuter who is using the route for private vehicles be charged the above toll. By charging such a toll, the travel externality is internalized and some of the commuters are discouraged from using the route in private vehicles, which increases the efficiency of the transportation system. With the toll at the optimal level, it is not difficult to prove that marginal social costs are the same for both routes,

$$SC_1 = SC_2 = \frac{bde - aR_1 - 2bF}{be - R_1} \tag{50}$$

4.3 Baress paradox

Before the causeway was added, each route (AUB or AWB) has half of the total traffic flow, and the total social cost was

$$TC_{withoutcauseway} = aF + \frac{F^2}{2e} \tag{51}$$

As previously specified, with the causeway added we have

$$C_1 = a + \frac{F_1 + F_3}{e}; \tag{52}$$

$$C_2 = a + \frac{F_2 + F_3}{e}; \tag{53}$$

$$C_3 = k + \frac{F_1 + F_3}{e} + \frac{F_2 + F_3}{e}; \tag{54}$$

$$F_1 + F_2 + F_3 = F. \tag{55}$$

Accordingly, the total cost can be written as a function of F_1 and F_2 after considering the flow constraint,

$$TC = C_1F_1 + C_2F_2 + C_3F_3 = (k + \frac{2F}{e})\, F - (k + \frac{2F}{e} - a)\, F_1 - (k + \frac{2F}{e} - a)\, F_2 + \frac{F_1^2}{e} + \frac{F_2^2}{e} \tag{56}$$

By minimizing the total cost with respect to F_1 and F_2, we obtain the following social optimal travel flow and the total social cost.

$$F_1 = F_2 = \frac{ke + 2F - ae}{2}; \qquad F_3 = ae - ke - F; \tag{57}$$

$$TC_{withoutcauseway} = \frac{2keF - (a-k)^2e^2 + 4(a-k)eF}{2e}. \tag{58}$$

Compared with the equilibrium traffic flow determined using the average private cost, F_3 is smaller under social optimization, suggesting that fewer travelers are using the causeway and the congested segments at the both ends. In addition, we can prove that the total social cost will be lower after the causeway is added, because

$$TC_{withoutcauseway} - TC_{withoutcauseway} = \frac{(ae - ke - F)}{2e} > 0. \tag{59}$$

Therefore, with socially optimal solutions, adding a causeway will reduce the total travel cost, indicating that the Braess paradox is resolved.

At the social optimum, the difference between social marginal cost and the average private cost, i.e., the traffic externality, will be $(F_1 + F_3)/e$ on AU and $(F_2 + F_3)/e$ on WB, with both evaluated at the socially optimal traffic flow. This determines the optimal toll and toll revenue on the congested roads (AU or WB), equal to

$$Toll = \frac{a-k}{2} \tag{60}$$

$$\mathrm{Re}venue = \frac{(a-k)^2e}{4} \tag{61}$$

Therefore, to attain a socially optimal solution and resolve the Braess traffic paradox, every commuter who is using the causeway should be charged a toll of the above amount, thus increasing the congestion of both segments of AU and WB. By charging such a toll, the travel externality is internalized and some of the commuters will be discouraged from using the causeway, making the transportation system more efficient. With the toll at the optimal level, it is not difficult to prove that the marginal social costs are the same for all three routes,

$$SC_1 = SC_2 = SC_3 = 2a - k \tag{62}$$

5. Conclusions

This paper illustrates that investment in highways and the expansion of routes does not always help to mitigate traffic congestion and reduce travel cost as intuition would suggest. Specifically, we present three cases in which highway expansion or construction is incapable of reducing congestion and may even increase travel costs by attracting more users from uncongested routes or public transit. First, as the Pigou-Knight-Downs paradox states, expanding the capacity of a bridge does not necessarily reduce travel cost. This occurs because traffic may simply shift to the bridge from uncongested routes, increasing the congestion on the bridge. Second, as shown by the Downs-Thompson paradox, increasing the capacity a route for private vehicles may actually make congestion worse. This occurs when commuters shift from public transport, which enjoys an economy of scale, to private cars that cause a negative traffic externality. Third, as the Braess paradox proves, adding extra capacity to a network, when the moving entities select their routes according to self interest, could in some cases increase the total commuting cost. In all three cases, commuters make decisions based on their private average cost, and in equilibrium all routes or modes have the same average travel cost.

The paper shows that the above traffic paradoxes can be resolved if the traffic externality is internalized and commuters are distributed by minimizing the total social travel cost. In other words, increasing road capacity reduces commuting cost when travel decisions are made based on the marginal social cost instead of the average private cost. To reach the social optimum, commuters who travel on congested routes need to be charged a toll. The toll is determined by the difference between the marginal social cost and the average private cost at the socially optimal traffic volume. For all three cases discussed above, this paper has derived the optimal tolls and toll revenues. At these optimal values, the marginal social costs are the same for different routes.

ACKNOWLEDGEMENTS. The authors thank Yiling Zhang for his assistance.

References

Arnott, R., & Small, K. A. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

Braess, C., Nagurney, A., & Wakolbinger, T. (2005). On a Paradox of Traffic Planning. *Transportation Science, 39*(4), 446-450.

Downs, A. (2004). *Still Stuck in Traffic: Coping with Peak-Hour Traffic Congestion*. Washington D.C.: Brookings Institution Press.

Hartman, J. L. (2007). A Route Choice Experiment With an Efficient Toll, Working Paper 1407, Department of Economics, University of California at Santa Barbara.

Murchland, J. (1970). Braess's Paradox of Traffic Flow. *Transportation Research, 4*, 391-394.